EngOpt 2008 - International Conference on Engineering Optimization
Rio de Janeiro, Brazil, 01 - 05 June 2008.

FORMULATIOS FOR THE OPTIMAL DESIGN OF RC WIND TURBINE TOWERS

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Abstract: The production of electrical energy from the wind energy is one of the most important projects to reduce the emissions of carbonic gas, preserve the environment and earn carbon credits. According to projections, around one thousand towers will be installed in Brazil from 2006 until 2010. We intend in this work to present some results of the development of software to minimize the cost of these kind structures. Several problems were proposed and solved. The objective of the formulations was to find optimal results taking into consideration the cost, computational time, construction techniques and precision of structural models. We considered RC towers subjected to dynamic wind loads and the effects of the vibration due to the components of the wind energy generator. The structure was discretized in finite elements and the concept effective stiffness of RC structures was used in one non-linear dynamic model to accomplish the structural analysis. The design variables are geometrical properties and reinforcement steel area of the structural elements. To compute the cost function we consider the costs of concrete and steel. Constraints related to stress, displacements and frequencies of vibration are applied. To solve the optimizations problems we used the augmented Lagrangian Method for dynamic structural problems. Directions for future studies are presented.

Keywords: Optimization, Wind Energy, Non-linear Dynamic

1. INTRODUCTION AND MOTIVATION

The optimization is an extremely relevant subject, because it treats, among other approaches, of the minimization of the cost of structures. In the present work, the examples include the design of large structures subjected to dynamic loading, more specifically, the reinforced concrete towers for supporting the wind turbine generators, here simply denoted by wind turbine towers (WTT).

The eolic energy, here also denoted as wind energy, is the one that is obtained from the winds. Eolo, in the Greek mythology, was the god of the winds. One of the cleanest ways to produce electrical energy is to convert it from the wind energy. The power stations to produce this kind of energy practically do not attack significantly the environment, except for the physical presence of the immense generating mills and other considerations that will be stated ahead in this work. In these days, the unitary cost of the wind energy is larger than the hydroelectric energy (about 50%), the ones which, however, they cause great impact in the environment. This cost can get lower if there are a good "critical mass" of investments of that type and the improvement of the designs of the structures, turning them more economical and efficient. Several Brazilian areas, according to Brazilian wind map (MME/ELETROBRÁS-2001), have a very good potential for the installation of wind farms (power stations to produce electrical energy from the wind energy).

The recent crisis of energy in Brazil revealed that the lack in the sector is potentially critic. The energy of the winds is an abundant source of renewable energy, clean and available in several places. The use of this energy source for the generation of electricity, in commercial scale, had beginning no more than 30 years ago and, using knowledge of the aeronautical industry, the equipments for generation developed quickly in terms of ideas and concepts for products of high technology. In the beginning of 70 decade, with the world crisis of the petroleum, there was a great interest of European countries and of The United States in developing equipments for production of electricity from others sources and helping to reduce the dependence of those countries from the petroleum and the coal. More than 50,000 new jobs were created and a solid industry of components and equipments were developed. Nowadays, the industry of wind turbines has accumulated annual growths larger than 30% and moving about 2 billion dollars in sales per year (data from 1999). There are more than 30,000 wind turbines of large load in operation in the world, with the installed capacity of the order of 13,500 MW. In the extent of the International Committee of Climatic Changes, the installation of 30,000 MW is being projected to the year 2030, being possible that this projection can be extended in function of the perspective of sallying "Carbon Certificates". In Denmark, the contribution of the wind energy is 12% of the total electric power produced; in the north of Germany (area of Schleswig Holstein) the contribution is larger than 16%; and the European Union has the goal of generating 10% of all electricity from the wind up to 2030.

In Brazil, although the use of the wind energy has been made traditionally to pump water, several studies to measure the wind speed have accomplished recently in several points of the national territory, they indicate the existence of an immense eolic potential still no explored. The capacity installed in Brazil is of 20.3 MW (data from 2004), with wind turbines of medium and large load connected to the national electrical net. Besides, dozens of wind turbines of small load exist working isolated of the conventional net for several applications – pumping, loading batteries, telecommunications and rural electrification.

As we told, the operation bases on the conversion of the wind kinetic energy (due to the movement of air masses through the atmosphere) to mechanical energy (rotation of the blades of the wind turbine rotor), which is transformed into electrical power by the electromagnetic generators. The blades of modern wind turbine rotors are aerodynamical equipments that have functions similar to the wings of airplanes. The tower is the element that supports the rotor and the blades at an appropriate level for the operation of the wind turbine (Fig. 1). The tower is a structural item of large weight of the system initial cost. In general, the towers are manufactured of steel (truss or tubular) or of reinforced concrete (RC). Wind turbines towers of small capacity are usually guided (supported by cables under tension) while the ones of medium and large capacities are cantilever beams, clamped in the foundation.

Winds with low speed do not have enough energy to move the blades. These start turning when the wind speed reaches a certain minimum value (cut in wind speed), which usually varies between 2.5 and 4.0 m/s. With the increase of the wind speed, the applied potency to the rotor axis increases gradually until reaching the machine nominal potency, what happens in a certain nominal wind speed, which usually varies between 9.5 m/s and 15.0 m/s. For wind speed larger than the nominal one, in several machines, the potency stays constant until the cut out wind speed, in which the machine should leave operation automatically to avoid that it suffers structural damages. For values upper than this wind speed the rotor brakes and stop turning. The available energy varies with the square of the wind speed. If the wind speed doubles, it represents an increase of four times in the energy generated. Besides the operation wind speed, the structural system should also be analyzed under the maximum wind speed, in agreement with NBR-6123 (ABNT, 1988) code. According to this code, in some areas of Brazil, the wind speed can reach 51 m/s, computed as the average on 3 s, that can be exceeded once in 50 years, at 10m above the ground level, in open and plane terrain.

With respect to the environmental impact, there are four items that should be appraised for installation of great load plants, to know:

- aesthetic - the turbines of great load are objects of great visibility (a turning rotor of several meters of diameter is visible many kilometers away) and they interfere significantly in the natural landscapes; therefore restrictions can exist for their installation in some areas (for instance, in tourist areas or areas of great natural beauty);
- noise - the turbines of great load produce significant audible noise, so it is necessary to check the local regulations for installation in residential neighborhoods; in most of modern turbines the noise level has been reduced;
- shadows and reflexes - the blades produce shadows and/or movable reflexes that are also undesirable in the residential areas; this problem is more evident in points of high latitudes, where the sun has lower position in the sky.
- birds - in wind farms can happen mortality of birds due to the impact with the blades (it is believed that the birds have difficulties to see the blades in movement), for that it is not advisable their installation in areas of migration of birds, reproduction areas, areas of environmental protection, etc.; the low speeds of rotation of the blades in the current equipments reduce this problem.

Besides the advantages and mentioned characteristics, the choice of the present theme is related also with the fact that it will be installed in Brazil, among the years of 2005 to 2008, 1.1 GW obtained from the wind energy (Wind Blatt, 2005), what is equivalent to approximately 1200 wind towers. It is intended, in the present work, to accomplish the optimal design of typical RC towers. In a moment when to get clean energy is extremely important for the planet sustainability, this work shows a possible direction to collaborate expressively with the improvement of the life quality in the planet and to reduce the global warming.

Fig. 1 – Wind turbine towers

2. A BRIEF REVIEW OF LITERATURE

As we stated before, the objective of this work is to describe some procedures for optimization of RC wind turbine towers (Fig. 1), subjected to a wind loading, based on experimental data and a non-linear dynamic analysis model. First, the non-linear dynamic model used in this work to accomplish structural analysis is presented. This model is based on experimental data and on the discrete dynamic model of NBR-6123 code (ABNT, 1988). In the model, the structure's effective stiffness is computed as a function of the bending moment in each step of the P-delta method. After that, the optimization problem is described, containing a description of the structure, the equations adopted to represent the effective stiffness, the formulation of the problem and the obtained results. Finally, conclusions based on the present study and suggestions for futures work are presented.

To solve the problems discussed here, use of advanced tools and models is needed, such as Lagrangian optimization, finite element method, non-linear dynamic analysis, reinforced concrete modeling and wind loading. The RC analysis is done based on the NBR-6118 code (ABNT, 2003). A review of the literature indicates that the effective stiffness of RC structures depends on the bending moment level, as well as the distribution and amount of reinforcement. An equation proposed by Branson (1963) for the calculation of the effective stiffness was incorporated in ACI-318 (ACI, 1971) and recently in NBR-6118 code (ABNT, 2003). Several researchers have used Branson's equation to compute the displacement of RC beams and slabs. Inspired in Branson's equation, Brasil and Silva (2006) developed a methodology for calculation of effective stiffness of RC beams subjected to bending moment and shear load. The methodology consists of using optimization techniques to minimize the error between the displacements measured in tests with those given by the integration of elastic line, and thus finding the stiffness that effectively works under a given loading. Some equations obtained by these authors will be used in the present work for calculation of the effective stiffness of the structural elements.

For the optimization process, the augmented Lagrangian method as described by Chahande and Arora (1994) and Arora (2004) is used. This method transforms a constrained optimization problem into an unconstrained optimization problem. The objective and constraints functions are combined with the Lagrange multipliers and penalty parameters to create the augmented Lagrangian functional. A sequence of functionals is created by properly altering the penalty parameters and Lagrange multipliers. The unconstrained minimum value of the functional in this sequence converges to the minimum of the constrained problem.

In the work of Silva and Brasil (2006) a non-linear dynamic model is presented for analysis of slender structures under dynamic wind loading. The model is based on NBR-6123 code (ABNT, 1988) and on the effective stiffness equations given by Brasil and Silva (2006). Once equations for the calculation of the effective stiffness are adopted, a non-linear static analysis of the structure under the mean wind speed loading (ABNT, 1988) is accomplished where, in each iteration of the P-delta method, the effective stiffness of RC elements is computed as a function of the bending moment level. Considering the effective stiffness obtained in the final iteration of the P-delta method, the authors calculate the natural frequencies and modes of vibration of the structure. These modes and frequencies are used to perform the floating wind analysis (ABNT, 1988) of the structure. These authors consider that the structure, under the wind loading, vibrates around an equilibrium configuration given by the P-delta method, and the amplitude of displacement is given by the dynamic wind analysis. Sum of the non-linear static analysis and the floating wind analysis results constitutes the non-linear dynamic analysis of the structure. Silva and Brasil (2006) concluded that the dynamic internal loads of the non-linear dynamic model are 15% larger than those with the linear dynamic model (ABNT, 1988). When the values obtained with the non-linear dynamic model are compared with those given by the linear static model of NBR-6123 code (ABNT, 1988), it is concluded that those are 50% larger than the linear static analysis results. This non-linear dynamic model is used in the present work.

In case of wind turbine towers, the IEC-61400-1 (IEC, 2005) code gives the rules and requirements for the design of wind turbine towers. A very practical approach is shown by Teixeira and Barros (2005), where they designed a foundation of a WTT. They accomplished a modal dynamic structural analysis, considering the contribution of the three first modes and showed that the contribution of the first mode is preponderant. The dynamic analysis accomplished by these authors is very similar to that one proposed by Silva and Brasil (2006) and which will be used in the present work. The main difference between these two approaches is that in the model of Silva and Brasil (2006) they included the geometrical and physical non-linearity, what do not happened with those authors that used linear model for the structural analysis. The solution for the foundation design adopted by Teixeira and Barros (2005) is an RC footing, while that the soil is modeled as elastoplastic springs. Negm and Maalawi (2000) accomplished a structural optimization of tubular steel WTT. They developed a simplified method to accomplish the structural analysis instead of the traditional finite element method (FEM) approach. The main conclusion of these authors is that the simplified method proposed by them saves more computational time than the FEM and other discrete methods. Madsen et al (1999) made a lot of tests and measurements of wind ultimate loads and the structure response. They concluded that because of the significant statistical variation of measured data is necessary a high number of simulation to obtain a precise estimate. Murtagh et al (2005) developed and analyzed a model with two degrees of freedom, including structural and soil stiffness. They concluded that coupled modes can cause damping of the structural system. There are several import works from other authors on this area and a more profound bibliographical review will be treated in other work

3. NON-LINEAR DYNAMIC ANALYSIS

3.1 Linear Static Analysis (LSA)

According to NBR-6123 code (ABNT, 1987), V_0 (*m/s*) is the mean wind speed computed based on 3 *s* interval, at 10 *m* above ground, for a plain terrain with no roughness, and a return period of 50 years. The topographic factor is S_1, while the terrain roughness factor is S_2, given as

$$S_2 = bF_r(z/10)^p \tag{1}$$

where *b*, *p* and F_r are factors which depend on the terrain characteristics, and *z* is the height above ground in meters. The statistical factor is S_3. Factors S_1, S_2 and S_3 are given in tables in the Brazilian Code NBR-6123 (ABNT, 1987). The characteristic wind speed (*m/s*) and the wind pressure (*Pa*) respectively, are

$$V_k = V_0 S_1 S_2 S_3 \quad \text{and} \quad q = 0.613 V_k^2 \,. \tag{2}$$

The wind load (*N*) on an area *A* (projection on a vertical plane of a given object area in m^2) is computed as

$$F = C_a A q \,, \tag{3}$$

where C_a is the aerodynamical coefficient. The Brazilian Code NBR-6123 (ABNT, 1987) presents tables for C_a values.

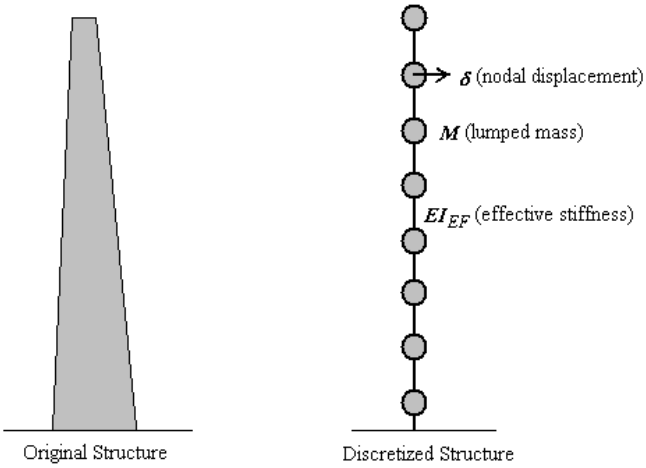

Fig. 2 - Typical RC WTT: original and discretized structure

2.2 Linear Dynamic Analysis (LDA)

If the first natural frequency of vibration of a given structure is smaller than 1 *Hz* (ABNT, 1987), it is necessary to proceed with the dynamic analysis of the structure. According to NBR-6123 code (ABNT, 1987), the dynamic analysis is performed as follows. For the *j*-th degree of freedom, the total load X_j due to direct wind along the tower is sum of the mean and the floating loads given as:

$$X_j = \overline{X}_j + \hat{X}_j \,. \tag{4}$$

The mean load $\overline{X}_j$ is given as

$$\overline{X}_j = \overline{q}_o b^2 C_j A_j \left(\frac{z_j}{z_r}\right)^{2p}, \tag{5}$$

where

$$\overline{q}_o = 0.613 \overline{V}_p^2 \quad \text{and} \quad \overline{V}_p = 0.69 V_0 S_1 S_3 \qquad (\overline{q}_o \text{ in } N/m^2 \text{ and } \overline{V}_p \text{ in } m/s), \tag{6}$$

and *b* and *p* are given in Table 20 of NBR-6123 code (ABNT, 1987); z_r is the level of reference, taken as 10 *m* in this work; $\overline{V}_p$ is the design wind speed corresponding to the mean speed during 10 minutes at 10 *m* above the ground level for a terrain roughness (S_2) for category II.

The floating component $\hat{X}_j$ in Eq. (4) is given as

$$\hat{X}_j = F_H \psi_j \varphi_j \tag{7}$$

where

$$\psi_j = \frac{m_j}{m_o}, \quad F_H = \overline{q}_o b^2 A_o \frac{\sum_{i=1}^{n} \beta_i \varphi_i}{\sum_{i=1}^{n} \psi_i \varphi_i^2} \xi, \qquad \beta_i = C_{ai} \frac{A_i}{A_o} \left(\frac{z_i}{z_r}\right)^p \tag{8}$$

and m_i, m_0, A_i, A_0, ξ and C_{ai}, respectively, are the lumped mass at the *i*-th degree of freedom, a reference mass, the equivalent area at the the *i*-th degree of freedom, a reference area, the dynamic coefficient of amplification in Figs. 14 to 18 of ABNT (1987), and the aerodynamical coefficient for area A_i.

Note that $\boldsymbol{\varphi} = [\varphi_i]$ is a given mode of vibration. To compute φ_i and ξ, it is necessary to consider the structural mass and stiffness. The lumped mass can be easily calculated by summing the mass around an influence region of the node. The total homogenized moment of inertia of the cross-section is given as

$$I_{\text{total}} = I_c + I_{s\,\text{hom}}, \quad I_{s\,\text{hom}} = I_s\left(\frac{E_s}{E_{c\,\text{sec}}} - 1\right), \; E_{c\text{sec}} = 0.9 \times 6600 \sqrt{f_{ck} + 3.5} \; (MPa), \tag{9}$$

where E_s, $E_{c\,\text{sec}}$, I_s, $I_{s\,\text{hom}}$, I_c and f_{ck}, respectively, are the elasticity modulus of steel, the secant elasticity modulus of concrete (ABNT, 1978), moment of inertia related to the structure axis of the total longitudinal steel area, the homogenized moment of inertia of the longitudinal steel area, the moment of inertia of the total cross-section and the characteristic compressive resistance in *MPa* for 28 days old concrete. Since this procedure is based on linear dynamic model, we consider the cross-section moment of inertia as the total moment of inertia:

$$I = I_{\text{total}}, \tag{10}$$

of each section to compute stiffness matrix of the structure. We consider that the structure under linear static behavior does not suffer any plastification or irreversible cracking, thus the stiffness to be considered must be the total stiffness.

Consider a given vector $\hat{Q}_i$ which represents a quantity, such as, internal loads, stress, strain, etc., due to the *i*-th natural mode of vibration. The contribution $\hat{Q}$ of *r* modes in the dynamic analysis is computed as

$$\hat{Q} = \left[\sum_{k=1}^{r} \hat{Q}_i^2\right]^{1/2}, \text{ while } Y_i = \frac{1}{3} X_i \tag{11}$$

is the transverse load, due to the variation of the wind direction.

2.3 Non-Linear Dynamic Analysis (NDA)

As stated before, loads due to the wind speed have two components, the static loads due to mean wind speed and the dynamic loads due to the floating wind speed. The static loads are computed using Eqs. (5) and (6). We call the first results obtained using these equations as the first order static internal loads. At this point, we consider that the structure under these static loads is subjected to the P-delta effect. The static displacements ($\bar{\delta}_{i(j)}$), at the *i*-th node and the *j*-th iteration of the P-delta method, are computed considering the effective stiffness. Differently from Section 2.2, we consider the following expressions to compute the moment of inertia (Brasil and Silva, 2006) at the *i*-th node and the *j*-th iteration of the P-delta method:

$$I_{i(j)} = I_{EF\ i(j)} = w_{i(j)} I_{\text{total}\ i},\ w_{i(j)} = w\left(x_{i(j)}\right) \quad \text{and} \quad x_{i(j)} = \frac{\overline{M}_{k\,i(j-1)}}{M_{ui}}, \tag{12}$$

where I_{EF}, w, x, $\overline{M}_k$ and M_u, respectively, are the effective moment of inertia, the parameter of effective stiffness, the level of strength, the working bending moment due to mean wind speed and the ultimate code based moment of a given cross-section. In Eqs. (12), effect of damaged cross-sections is accounted by the effective stiffness concept.

Finally, the P-delta effect is computed, at the *i*-th node and the *j*-th iteration of the P-delta method, as

$$\Delta\overline{M}_{k\,i(j)} = \Delta N_{k\,i}\,(\bar{\delta}_{i(j)} - \bar{\delta}_{i(j-1)}) \text{ and } \overline{M}_{k\,i(j)} = \overline{M}_{k\,i(j-1)} + \sum_{l} \Delta\overline{M}_{k\,l\,(j)} \tag{13}$$

where ΔN_k is the nodal characteristic axial load. We call the final results obtained using these equations as the second order static internal loads. Considering the stiffness obtained in the final iteration of P-delta method, we compute the mode shapes and frequencies of vibration of the structure and so accomplish the dynamic analysis, as described in Eqs. (7) and (8). We considered that the structure displaces around the equilibrium position given by the P-delta method.

4. OPTIMIZATION OF RC WIND TURBINE TOWERS

4.1 Structure Characteristics

In this work, we considered real data from wind turbines available in the market. We adopted the characteristics of the V80-2.0 MW made by Vestas SA. The RC structures analyzed here have H = 65, 82.5 and 100 *m* tall having circular cross-section. The diameter, thickness and steel areas change along the height of the tower. The concrete used in the fabrication of the tower has a characteristic resistance at 28 days old (f_{ck}) as 45 *MPa*, which gives secant elasticity modulus (E_{csec}) of 41.4 *GPa* from Eq. (9). This value is relatively high for the concrete in question and also different from that given by the new NBR-6118 code (ANBT, 2003); however the equations presented by Brasil and Silva (2006) were developed using Eq. (9) for the calculation of the elasticity modulus. Brasil and Silva (2006) presented important comments on the influence of the concrete elasticity modulus on the calculation of the effective stiffness. Applying the safety factor, the concrete design resistance is then f_{cd} = 45/1.3 *MPa*. The steel has: cover of 25 *mm*, design resistance of f_{yd} = 500/1.15 *MPa* and elasticity modulus E_s = 210 *GPa*. The unit costs of concrete (c_c) and steel (c_s) are respectively US\$ 82.61/m^3 and US\$ 1.13/*kg*.

The structure, similar to that shown in Fig. 2. is discretized with 41 nodes and 40 elements, and the first element starts at the first node and ends at the second, the second element starts at the second node and ends at the third, and so on. With this discretization, the structure has 240 degrees of freedom. The displacement vector corresponding to the structural degrees of freedom, is also denoted as the state variable vector.

We consider the basic wind speed of V_0 = 45 *m/s*, the topographic factor is S_1=1, terrain roughness category IV, class B, which gives $S_2 \equiv (b; p; Fr)$, and S_3=1.1 is the statistical factor. As we stated before, the wind load on an area A is $F = C_a A q$, where

C_a is the aerodynamical coefficients. Some equipment are installed on the structure, such as the turbine, the blades, internal stairway, night signer lights and system of protection against atmospheric electrical rays. The values of A and C_a for equipments are:

- tower, $0 \le z \le H$, A = "variable with the design" and $C_a = 0.6$;
- turbine and blades, $z = H$, $A = 120\ m^2$ and $C_a = 1$.

Besides these areas and aerodynamical coefficients, the tower mass is considered distributed along the structure proportionally to the volume and it can be computed using a density of 2500 kg/m^3. The masses of the other components are considered as 40 kg at each node, with the exception of the top node, node number 40, where a lumped mass of 104000 kg is considered due to the rotor and blades masses. The blades are subjected to different wind pressures and inertial loads along the axis z, so because of that, the loads related to the rotor and blades are considered with an eccentricity of 11 m above H.

Based on the results obtained by Brasil and Silva (2006), we adopt the following equation for the effective stiffness parameters:

$$w(x) = -1.5\,x^3 + 3.3\,x^2 - 2.5\,x + 1.1 \tag{14}$$

We add the following constraints to the Eq. (14):

$$w_s \le w \le 1, \quad \text{for} \quad i = 0, 1, \ldots, n \tag{15}$$

Where

$$w_s = I_s / I_{\text{total}}, \tag{16}$$

and n+1 is the number nodes. Note that the maximum value (upper limit) of w is 1 and the minimum value (lower limit) varies as a function, amount and distribution of longitudinal of the geometry reinforcement.

4.2 The Optimization Problem

The problem consists of minimizing the costs of the concrete and steel used in the construction of a structure similar to those shown in Fig.s 1 and 2, and described in the Section 3.1. This is a cantilever structure clamped at the ground level.

The first formulation proposed in this work considered 164 design variables to solve the problem; the design variable vector is $\boldsymbol{b}^t = [ø_0\ e_0\ A_{s0}\ A_{sw0}\ ø_1\ e_1\ A_{s1}\ A_{sw1} \ldots ø_{40}\ e_{40}\ A_{s40}\ A_{sw40}]$, where $ø_i$, e_i, A_{si} and A_{swi} are respectively the external diameter, the thickness, the longitudinal steel area and the transverse steel area at the i-th node of the structure.

The optimization problem is then to minimize the cost function:

$$f(\boldsymbol{b}) = C_c + C_s \tag{17}$$

where C_c and C_s are respectively the costs of the concrete and steel. They can be written:

$$C_c = V_c\, c_c \tag{18}$$

$$C_s = M_s\, c_s \tag{19}$$

where V_c is the total concrete volume and M_s is the total steel mass.

Constraints for the problem are:

- the minimum value for the longitudinal steel area at each node i

$$-A_{s\,i} + 0.15\% A_{ci} \le 0\,;\ i=0,\ldots,40 \tag{20}$$

- the minimum value for the traverse steel area armor at each node i

$$-A_{sw\,i} + \frac{0.07\% A_{ci}}{u_i} \le 0\,;\ i=0,\ldots,40 \tag{21}$$

- the cross-section resistance to axial loading and bending moment at each node i

$$M_{di} - M_{ui}(N_{di}) \le 0\,;\ i=0,\ldots,40 \tag{22}$$

- the cross-section resistance to the shear loading at each node i

$$Q_{di} - Q_{ui} \le 0\,;\ i=0,\ldots,40 \tag{23}$$

- the minimum thickness at each node i

$$-e_i + 8.8\,cm \le 0\,;\ i=0,\ldots,40 \tag{24}$$

- the compatibility between the thickness and diameter at each node i

$$2e_i - ø_i \leq 0 \text{; } i=0,...,40 \tag{25}$$

- the consideration that all design variables must be positive

$$-b_i \leq 0 \text{; } i=0,...,163 \tag{26}$$

- the first frequency of vibration must be larger than a minimum

$$-f_1 + 0.475Hz \leq 0 \text{;} \tag{27}$$

In Eqs. (20) and (21), A_c is the concrete area at a given cross-section and u the external perimeter of the torsion nucleus. In the Eq. (22), M_d is the design bending moment, calculated as $\gamma_f M_k$, where γ_f is a load factor, while N_d is the design axial load, computed as $\gamma_f N_k$, where N_k is the characteristic axial load. In Eq. (23), Q_d is the design shear load, calculated as $\gamma_f Q_k$, where Q_k, is the characteristic shear load and Q_u is the ultimate code based shear load, which is a function of the traverse reinforcement of the given cross-section. The values adopted for γ_f is 1.4 for bending moment and shear load, and 0.9 for compressive axial load. The equation (27) is one of the most import constraints and the lower limit of the main frequency must be defined in function of the rotor operation speed. With these definitions, the optimization problem proposed in Eq. (18) to (27) presents 164 design variables, 411 constraints and 240 degrees of freedom.

One can note in the formulation presented above that the diameters and thickness can assume any value in a certain level z. If we assume that the WTT will present:

- constant taper (variation of the diameter in function of the length);
- constant thickness and the thickness is adopted as the minimum available, usually in function of constructive constraints;
- the steel area is computed in function of the internal loads;

we reduce drastically the number of design variables. So the design variable vector to solve the problem now can be defined as $\boldsymbol{b}^t = [ø_t \; t]$, where $ø_t$ is the tip diameter (top of the structure diameter) and t is the taper. The diameter in a given section of level z_i can be computed as

$$\phi_i = \phi_t + (H - z_i)t \, . \tag{28}$$

Other important consideration is the computation of steel area is in function of the internal loads. The internal loads in a non-linear dynamic analysis are function of the structure effective stiffness, which is function of the steel area (Eq. 12). So we have a cyclical reference related the steel areas that can be solved by iterative methods. With these definitions, this last optimization problem presents 2 design variables, and the same number of constraints and degrees of freedom. In this case, with just a few design variables and the steel areas being computed exactly in function of the internal loads, the cost can be optimized much more precisely with less computational time.

Several other optimization problems were developed by the authors and a precise description of these formulations will be treated in other paper. Comparing the results the authors concluded that this last formulation is the one that present the minimum cost and computational time, besides of that in terms of constructive techniques, this last formulation is very efficient.

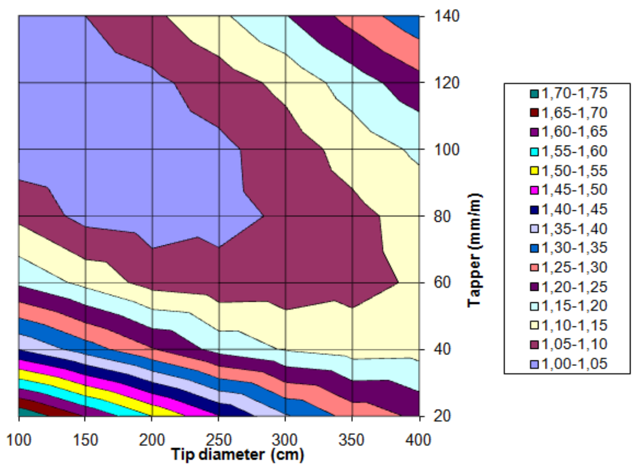

Fig. 3 – Non-dimensional cost in function of the taper and the tip diameter

4.3 Results Obtained

The optimization problems defined by Eq. (18) to (28), beside others, were solved and the results are shown in Fig.s 3 and 4. The authors organized the results related to cost in Fig. 3, where the non-dimensional cost is presented as a function of the tip diameter and the taper. The non-dimensional cost is defined as the cost divided by a reference value. In this work the reference value is the minimum obtained for a given height. We noted that Fig. 3 has the same shape independent of the tower height.

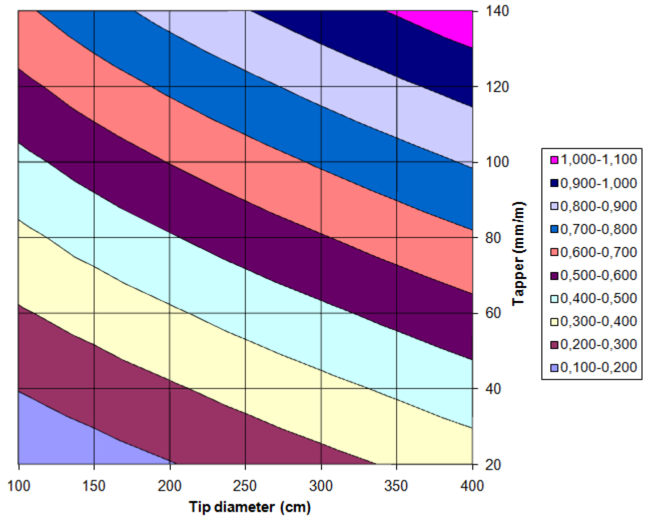

Fig. 4 – First frequency of vibration (*Hz*) of 82.5 m tall tower in function of the taper and the tip diameter

In Fig. 4 is shown the first frequency of vibration for a 82.5 *m* tall tower in function of the taper and the tip diameter. For different heights the following correction factor may be used

$$\chi = -0.01H + 1.825, \qquad (29)$$

where *H* must be used in meters. Fig. 4, besides of Eq. (29), are very important in practice because with that we can escape of non-desirable frequencies, specially those that may cause resonance.

It is important to note that the base of the structure is considered clamped in the ground level. However, the foundations are not infinitely rigid and they influence the natural frequencies of vibration of these structures. Silva and Brazil (2003) accomplished the optimization a telecommunication tower with a formulation that is quite different from the one presented here. They showed that the flexibility of the foundation can drastically affect the calculation of the dynamic response of the towers subjected to dynamic wind loading. To perform an integrated analysis of the foundation and structure is extremely important for a more reliable dynamic analysis (Silva et al, 2002).

5. CONCLUSIONS

A non-linear dynamic model based on experimental data for structural analysis of RC slender structures was presented. This model was used to formulate some optimization problems to minimize the cost of RC wind turbine towers. Optimization problems were formulated and towers with different heights were analyzed to obtain the best formulation in terms of cost, precision and computational time. The first optimization problem presented here considered, such others, the diameter and the steel area in each section of a given discretized structure as design variables. Te second one considered the diameter in function of the tip diameter and the tape, and the steel area as a function of the internal loads and not like design variables, as formulated first. The main conclusion of this work is that constant taper and the steel area computed in function of the internal loads lead to minimum cost and computational time. Two graphs were shown giving the relation between the cost and the frequency of vibration with the tip diameter and the taper. We suggest for further work the development of other paper describing in details the several optimization problems proposed and solved by the authors, as the optimization and mathematical techniques used to obtain Eq. (29) and Fig.s (3) and (4).

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ACKNOWLEDGMENTS

This work is result of research projects of Post-Doctorate accomplished by Marcelo Araujo da Silva. The work was carried out at The University of Sao Paulo, in Sao Paulo-SP., Brazil, under the supervision of Prof. Dr. Reyolando M. L. R. F. Brasil, and at The University of Iowa, in Iowa City-IA, USA, under the supervision of Prof. Dr. Jasbir S. Arora. The programs were sponsored by FAPESP under the Projects N.º 2005/02815-5 and 2006/04744-0. All these supports are gratefully acknowledged.